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Relations and Functions

Teaching-Learning Points

- Let A and B are two non empty sets then a relation from set A to set B is defined as R = {(*a*.*b*) : $a \in$ A and $b \in$ B}. If (*a*.*b*) ∈ R, we say that a is related to *b* under the relation R and we write as *a* R *b*.
- R $R \subseteq A \times B$.
- A relation R in a set A is a subset of A × A.

  Types of relations :

  (*i*) empty relation : $R = \phi C A \times A$

  (*ii*) Universal relation $R = A \times A$

  (*iii*) Reflexive relation : $(a, a) \in R \; \forall \in$ A.

  (*iv*) Symmetric relation : If $(a, b) \in R \Rightarrow (b, a) \in R \quad \forall\, a, b \in A$.

  (*v*) Transitive relation : If $(a, b) \in R$ and

  $\Rightarrow \; (a, c) \in R \quad \forall\, a, b, c \in A$.
- A relation R is set A is said to be equivalence relation. If R is reflexive, symmetric and transitive.
- Let R is an equivalence relation is set A and R divides A into mutually disjoint subset A called partitions or subdivisions of A subsfying the conditions :

  (*i*) all element of A_i are related to each other, ∀ *i*.

  (*ii*) no element of A_i is related to any element of A_j, it *j*

  (*iii*) $UA_i = A$ and $A_i \cap$ Aj = ϕ, ≠ *j*.
- Type of Functions :

  (*i*) one-one (or injective) function : Let F : A → B, then for every $x_1, x_2 \in A$, $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.

  (*ii*) onto (or surjective function) : Let F : A → B, then for every $y \in$ B, there exists an element $x \in$ A such that $f(x) = y$.

  (*iii*) A function which is not one-one is called many-one function.
- A function which is not onto is called into function.
- A function which is both one-one and onto is called a bijective function.
- Let A be a finite set then an injective function F : A → A is subjective and conversely.
- Let F : A → B and g : B → C be two functions. Then the composition of F and g, denoted as gof is defined as the function gof : A → C given by g of (*x*) = *g* [*f*(*x*)]

  $\forall\, x \in$ A

- Composition of functions need not to be commutative and associative.
- If $F : A \to B$ and $g : B \to C$ be one-one (or on to) functions, then $gof : A \to c$ is also one-one (or on to) but converse is not true.
- A function $F : A \to B$ is said to be invertible if there exists another function $g : B \to A$ such that $gof = I_A$ and $fog = I_B$. The function g is called the inverse of the function F.
- A function $F : A \to B$ is said to be invertible if and only if F is one-one and onto (i.e. bijective).
- If $F : A \to B$ and $g : B \to C$ are invertible functions, then $gof : A \to C$ is also invertible and $(gof)^{-1} = F^{-1} og^{-1}$.

Binary operations :

- A binary operation * on a set A is a function $* A \times A \to A$ we denoted $*(a, b)$ by $a * b$.
- A binary operation * on a set A is called commutative if $a * b = b * a \ \forall \ a, b \in A$.
- A binary operation * on a set A is said to be associative if $a * (b * b) * c \ \forall \ a, b, c \in A$.
- The element $e \in A$, if it exists, is called identity element for binary operation * if $a * e = a = e * a$ $\forall a \in A$.
- The element $a \in A$ is said to be invertible with respect to the binary operation * if there exile $b \in A$ such that $a * b = e = b * a$. The element b is called morse of a and is denoted as a^{-1}.

Question for Practice

Evaluate the following Integrals

Very Short Answer Type Questions (1 Mark)

Q1. Let R be a relation on A defined as $R = \{(a, b) \in A \times A : a$ is a husband of $b\}$ can we say R is symmetric? Explain your answer.

Q2. Let $A = \{a, b, c\}$ and R is a relation on A given by $R = \{(a, a), (a, b), (a, c), (b, a), (c, c)\}$. Is R symmetric? Give reasons.

Q3. Let $R = \{(a, b), (c, d), (e, f)\}$, write R^{-1}.

Q4. Let L be the set of are straight lines in a given plane and $R = \{(x, y) : x \perp y \ \forall \ x, y \in L\}$. Can we say that R is transitive? Give reasons.

Q5. The relation R in a set $A = \{x : x \in z$ and $0 \le x \le 12\}$ is given by $R = \{(a, b) : |a - b|$ is a multiple if $4\}$ is an equivalence relation. Find the equivalence class related to {3}.

Q6. Let R_1 be the relation on R defined as $R = \{(a, b) : a \le b^2\}$. Can we say that R is reflexive? Give reasons.

Q7. Let R $\{(a, b) : a, b \in Z$ (Integers) and $|a - b| \le 5\}$. Can we say that R is transitive? Give reason.

Q8. If $A = \{2, 3, 4, 5\}$, then write the relation R on A, where $R = \{(a, b) : a + b = 6\}$.

Q9. If A {1, 2}, and $B = \{a, b, c\}$, then what is the number of relations on $A \times B$?

Q10. State reason for the relation R in the set {1, 2, 3} given by R {(1, 2), (2, 1)} not to be transitive.

Q11. If f is invertible function, find the inverse of $f(x) = \frac{3x - 2}{5}$.

Q12. If $f(x) = x + 7$ and $g(x) = x - 7$, $x \in$ R, find fog (x).

Q13. Write the inverse of the function $f(x) = 5x + 7$, $x \in$ R.

Q14. Show that f : R $\rightarrow$ R defined as $f(x) = x^2 + 1$ is not one-one.

Q15. Show that the function f : N $\rightarrow$ N defined by $f(x) = 3x$ is not an onto function.

Q16. Let * be a binary operation on Z defiand by $a * b = 2a + b - 3$, find 3 * 4.

Q17. Let * be a binary operation on N defined by $a * b = a^2 + b$ and O be a binary operation on N defined by $aob = 3a - b$ find (2 * 1) 02.

Q18. let * be a binary operation on R defined by $a * b = a - b$. Show * is not commutative on R.

Q19. Let * be a binary operation on N given by $a * b$ = l.c.m (a, b), $a, b \in$ N find (2 * 3) * 6.

Q20. Can we say that division is a binary operation on R? Give reasons.

Q21. Show that * : R × R $\rightarrow$ R given by $a * b = a + 2b$ is not associative.

Q22. Explain that addition operation on N does not have any identity.

Q23. What is inverse of the element 2 for addition operation on R?

Q24. Let * be the binary operation on N given by l.c.m $(a : b)$ find the identify element for * on N.

Q25. Let * be the binary operation on N defined by $a * b$ = HCF (a, b). Deos there exist identify element for * on N?

Short Answer Type Questions (4 Marks)

Q26. Show that f : N $\rightarrow$ N givne by

$$f(x) = \begin{cases} x + 1 \text{ if x is odd} \\ x - 1 \text{ is x is even, is bijective} \end{cases}$$

Q27. Let * be a binary operation on the set A = {0, 1, 2, 3, 4, 5} as

$$a * b = \begin{cases} a + b \text{ if } a + b < 6 \\ a + b - 6 \text{ if } a + b \geq 6, \end{cases}$$

Show that O is the identify element for this operation and each element a of the set is invertible with $6 - a$ being the inverse of a.

Q28. Let N be the set of all natural numbers and R be a relation on N × N, defined by (a, b) R $(c, d) \Rightarrow ad = bc$ $\forall$ $(a, b), (c, d) \in$ N × N. Show that R is an equivalence relation.

Q29. Let f : R $\rightarrow$ R be defined by $f(x) = 3x + 2$. Show that f is invertible. Find f : R $\rightarrow$ R.

Q30. Let * be a binary operation on N × N defined by $(a, b) * (c, d) = (a + c, b + d)$. Show that * is commutative as well as associative. Find the identity element for * on N × N if any.

Q31. Let T is a set of all triangles in a plane and R be a relation as R : T $\rightarrow$ T = $\{(\Delta_1, \Delta_2) : \Delta_1 \cong \Delta_2 \ \forall \ \Delta_1, \Delta_2 \in T\}$. Show that R is an equivalence relation.

Q32. Let * be the binary operation on Q (Rational numbers) defiend by $a * b = |a - b|$, show that

(*i*) * is commutative

(*ii*) * is not associative

(*iii*) * does not have identity element

Q33. Show that $f : R \to R$ defined by $f(x) = x^3 - 1$; is invertible. Find $f(x)$.

Q34. Show that if $f : B \to A$ is defined by $f(x) = \frac{3x+4}{5x-7}$ and $g : A \to B$ is defined by $g(x) = \frac{7x+1}{5x-3}$, then Fog = I_A and gof = I_B, where $A = R - \left\{\frac{3}{5}\right\}$ and $B = R - \left\{\frac{7}{5}\right\}$.

Q35. Show that the function $F : Q - \{3\} \to Q$, given by $F(x) = \frac{2x+3}{x-3}$ is not a bijective function.

Answers

Very Short Answer (1 Mark)

1. No, if is a husband of b, then b being a female can not be husband of anybody.
2. No, because $(a, c) \in R$ but $(c, a) \notin R$.
3. $R^{-1} = \{(b, a), (d, c), (f, e)\}$
4. No, If $x \perp y$ & $y \perp z \Rightarrow x \parallel z$.
5. {3, 7, 11}.
6. No, example $\frac{1}{3} \not\leq \left(\frac{1}{3}\right)^2$.
7. No, Let $a = 5$, $b = 10$, $c = 12$, then $(a, b) \in R$, $(b, c) \in R$ but $(a, c) \notin R$.
8. $R = \{(2,4), (3,3), (4,2)\}$
9. 64
10. $(1,1) \notin R$.
11. $f^{-1}(x) = \frac{5x+2}{3}$
12. x
13. $\frac{x-7}{5}$

16. 7

17. 13

19. 6

20. No, because Number divided by 0 does not belong to R.

21. Let $a = 2$, $b = 5$, $c = 8$, $(a * b) * c = (2 + 2 \times 5) * 8 = 12 * 8$

$= 12 + 2 \times 8 = 28$ and $a * (b * c) = 2 * (5*) = 2 * (5 + 2 \times 8)$

$= 2 * 21 = 2 + 2 \times 21 = 44$.

22. Because O + number = Number but O does not belong to N.

23. –2

24. 1

25. No

Very Short Answer (4 Mark)

29. $f^{-1} = (x) = \frac{x-2}{3}$

30. Identity does not exist

33. $f^{-1}(x) = (x+1)^{1/3}$

35. $f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \Rightarrow$ is one-one.

Let $y \in$ codomain then $f(x) = y$

$\Rightarrow$ $x = \frac{-3-3y}{2-y} \notin Q-\{3\}$ for some $y \in Q$

Example $2 \in$ codomain but

$= \frac{-3-3\times 2}{2-2}$ = Not defined, does not belong to domain