

Mathematic

MM.40

Note: Question 1 to 10 carry 4 marks each.

1. If $\sin(A + B + C) = 1$, $\cos(B + C) = \frac{1}{2}$ and $\sin B = \frac{\sqrt{3}}{2}$, find the value of A, B and C.
2. Evaluate:
$$\frac{5 \sin^2 30^\circ + \cos^2 45^\circ - 4 \tan^2 30^\circ}{2 \sin 30^\circ \cos 30^\circ + \tan 45^\circ}.$$
3. If $\frac{\cos^2 20^\circ + \cos^2 70^\circ}{2(\sin^2 59^\circ + \sin^2 31^\circ)} = \frac{2}{k}$, find the value of k .
4. Prove that: $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}.$
5. If $\sec 4A = \operatorname{cosec}(A - 20^\circ)$, find the value of A.
6. If $\tan A + \cot A = 2$, find the value of $\tan^2 A + \cot^2 A$.
7. In triangle ABC, right angled at B, if $\tan A = \frac{1}{\sqrt{3}}$, find the value of $\sin A \cdot \cos C + \cos A \cdot \sin C$.
8. Evaluate:
$$\frac{1}{\sin^3 30^\circ} + \frac{1}{\cos^2 30^\circ} - \frac{1}{2} \tan^2 45^\circ - \sin^2 90^\circ.$$
9. Prove that:
$$\sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \sin A.$$
10. Prove that:
$$\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \operatorname{cosec} \theta.$$