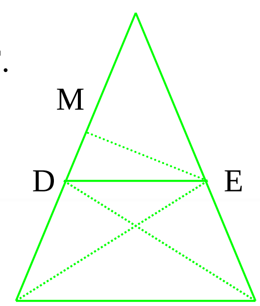
Mathematics

(Theorems)

If a line is drawn parallel to one side of a triangle, to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.



Given :In ΔABC, DE ∥ BC .

To Prove :AD/DB = AE/EC.

Construction :Draw CM ⊥ AB and join BE and CF.

Proof : $\frac{ar(\Delta ADE)}{ar(\Delta DBE)} = \frac{½.AD.EM}{½.DB.EM}$

$= \frac{AD}{DB}$ ………………(1)

Similarly $\frac{ar(\Delta ADE)}{ar(\Delta DCE)} = \frac{AE}{EC}$ …….(2)

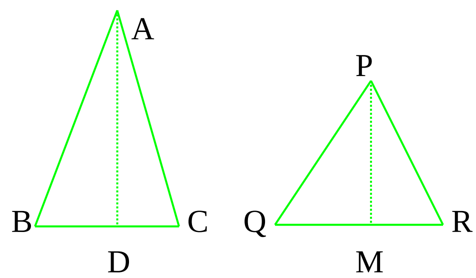
Now ar(ΔDBE) = ar(ΔDCE)…….(3)

[∵The area of triangles at the same base and between same parallel lines are equal]

From (1), (2) and (3)

$\frac{AD}{DB} = \frac{AE}{EC}$

The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.



Given : $\Delta ABC \sim \Delta PQR$

To Prove : $\dfrac{ar(\Delta ABC)}{ar(\Delta PQR)} = \dfrac{AB^2}{PQ^2} = \dfrac{BC^2}{QR^2} = \dfrac{CA^2}{RP^2}$

Construction : Draw $AD \perp BC$ and $PM \perp QR$.

Proof : $\dfrac{ar(\Delta ABC)}{Ar(\Delta PQR)} = \dfrac{\frac{1}{2}BC.AD}{\frac{1}{2}QR.PM}$

$= \dfrac{BC}{QR.} \times \dfrac{AD}{PM}$……….(1)

Now in ΔABD and ΔPQM

$\angle B = \angle Q$ [$\because \Delta ABC \sim \Delta PQR$]

$\angle ADB = \angle PMQ$ [$\because$ Each of 90^0]

$\therefore$ $\Delta ADB \sim \Delta PMQ$ [By AA similarity]

$\therefore$ $\dfrac{AD}{PM} = \dfrac{AB}{PQ}$ [CPST] ………..(2)

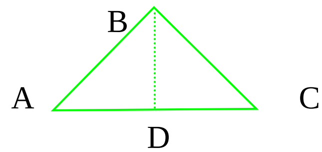
By (1) and (2)

$\dfrac{ar(\Delta ABC)}{ar(\Delta PQR)} = \dfrac{BC}{QR.} \times \dfrac{BC}{QR} = \dfrac{BC^2}{QR^2}$

But $\dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{AC}{PR}$ [$\because \Delta ABC \sim \Delta PQR$]

$\therefore$ $\dfrac{ar(\Delta ABC)}{ar(\Delta PQR)} = \dfrac{AB^2}{PQ^2} = \dfrac{BC^2}{QR^2} = \dfrac{CA^2}{RP^2}$

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.



Given: In ΔABC, $\angle B = 90$.

To Prove: $AB^2 + BC^2 = AC^2$.

Construction: Draw $BD \perp AC$.

Proof: In ΔABC and ΔABD

$\angle A = \angle A$ [Common]

$\angle ABC = \angle ADB$ [$\because$ Each of 90^0]

$\Delta ABC \sim \Delta ADB$ [$\because$ AA Similarity]

$\therefore \quad \dfrac{AB}{AD} = \dfrac{AC}{AB}$ [By C.P.C.T.]

$\Rightarrow \quad AB^2 = AD.AC$ ………………(1)

Similarly

$BC^2 = DC.AC$ ………………(2)

Adding (1) and (2)

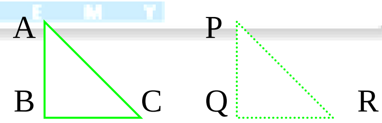
$AB^2 + BC^2 = AD.AC + DC.AC$

$= AC\,(AD + DC)$

$= AC.AC$

$\Rightarrow \quad AB^2 + BC^2 = AC^2$

In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.



Given: In ΔABC, $AB^2 + BC^2 = AC^2$.

To Prove: $\angle B = 90^0$.

Construction: Construct a ΔPQR such that AB=PQ, BC= QR and $\angle Q = 90^0$.

Proof: In ΔPQR, by Pythagorus theorem,

$PR^2 = PQ^2 + QR^2$.

$= AB^2 + BC^2$. [By construction]

$= AC^2$. [$\because AB^2 + BC^2 = AC^2$.]

$\therefore \quad PR = AC$.

Now by SSS property

$\Delta ABC \cong \Delta PQR$ [$\because$ AB=PQ, BC=QR & AC=PR]

$\Rightarrow \quad \angle B = \angle Q = 90^0$.
