Physics

NCERT Exemplar Problems

Chapter 6

Electromagnetic Induction

Answers

6.1 (c)

6.2 (b)

6.3 (a)

6.4 (d)

6.5 (a)

6.6 (b)

6.7 (a), (b), (d)

6.8 (a), (b), (c)

6.9 (a), (d)

6.10 (b), (c)

6.11 No part of the wire is moving and so motional e.m.f. is zero. The magnet is stationary and hence the magnetic field does not change with time. This means no electromotive force is produced and hence no current will flow in the circuit.

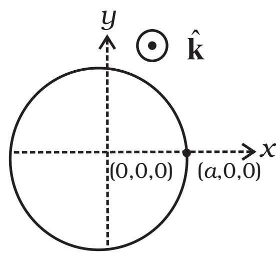
6.12 The current will increase. As the wires are pulled apart the flux will leak through the gaps. Lenz's law demands that induced e.m.f. resist this decrease, which can be done by an increase in current.

6.13 The current will decrease. As the iron core is inserted in the solenoid, the magnetic field increases and the flux increases. Lent's law implies that induced e.m.f. should resist this increase, which can be achieved by a decrease in current.

6.14 No flux was passing through the metal ring initially. When the current is switched on, flux passes through the ring. According to Lenz's law this increase will be resisted and this can happen if the ring moves away from the solenoid. One can analyse this in more detail (Fig 6.5). If the current in the solenoid is as shown, the flux (downward) increases and this will cause a counterclockwise current (as seen form the top in the ring). As the flow of current is in the opposite direction to that in the solenoid, they will repel each other and the ring will move upward.

6.15 When the current in the solenoid decreases a current flows in the same direction in the metal ring as in the solenoid. Thus there will be a downward force. This means the ring will remain on the cardboard. The upward reaction of the cardboard on the ring will increase.

6.16 For the magnet, eddy currents are produced in the metallic pipe. These currents will oppose the motion of the magnet. Therefore magnet's downward acceleration will be less than the acceleration due to gravity *g*. On the other hand, an unmagnetised iron bar will not produce eddy currents and will fall with an acceleration *g*. Thus the magnet will take more time.



6.17 Flux through the ring

$$\phi = B_o(\pi a^2)\cos\omega t$$

$$\varepsilon = B(\pi a^2)\omega\sin\omega t$$

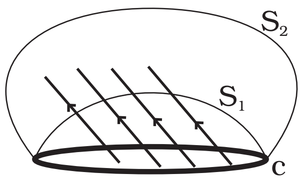
$$I = B(\pi a^2)\omega\sin\omega t / R$$

Current at

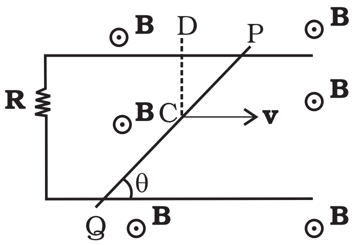
$$t = \frac{\pi}{2\omega};\ I = \frac{B(\pi a^2)\omega}{R} \text{ along } \hat{\mathbf{j}}$$

$$t = \frac{\pi}{\omega};\ I = 0$$

$$t = \frac{3}{2}\frac{\pi}{\omega};\ I = \frac{B(\pi a^2)\omega}{R} \text{ along } -\hat{\mathbf{j}}.$$



6.18 One gets the same answer for flux. Flux can be throught of as the number of magnetic field lines passing through the surface (we draw d$N = B\,\Delta A$ lines in an area Δ A ⊥to **B**), As lines of of **B** cannot end or start in space (they form closed loops) number of lines passing through surface S_1 must be the same as the number of lines passing through the surface S_2.



6.19 Motional electric field *E* along the dotted line CD (⊥ to both **v** and **B** and along **v** × **B**) = vB

E.M.F. along PQ = (length PQ)×(Field along PQ)

$$= \frac{d}{\cos\theta} \times vB\cos\theta = dvB.$$

Therefore,

$$I = \frac{dvB}{R}$$ and is independent of q.

6.20 Maximum rate of change of current is in AB. So maximum back emf will be obtained between 5s < t <10s.

If $u = L\,1/5\left(\text{for}\, t = 3\,\text{s}, \frac{dI}{dt} = 1/5\right)$ (L is a constant)

For 5s < t < 10s $u_1 = -L\frac{3}{5} = -\frac{3}{5}L = -3e$

Thus at t = 7 s, $u_1 = -3\,e$.

For 10s < t < 30s

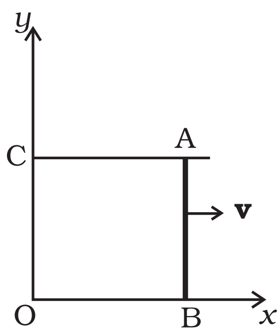
$$u_2 = L\frac{2}{20} = \frac{L}{10} = \frac{1}{2}e$$

For t > 30s $u_2 = 0$

6.21 Mutual inductance $= \frac{10^{-2}}{2} = 5\text{mH}$

Flux $= 5\times10^{-3}\times1 = 5\times10^{-3}$ Wb.

6.22 Let us assume that the parallel wires at are y = 0 and y = d. At t = 0, AB has x=0 and moves with a velocity $v\hat{\mathbf{i}}$.



At time t, wire is at $x(t) = vt$.

Motional e.m.f. $= (B_o \sin\omega t)vd(-\hat{\mathbf{j}})$

E.m.f due to change in field (along OBAC)

$$= -B_o\omega\cos\omega t\, x(t)d$$

Total e.m.f $= -B_o d\left[\omega x\cos(\omega t) + v\sin(\omega t)\right]$

Along OBAC, Current (clockwise) $= \frac{B_o d}{R}(\omega x\cos\omega t + v\sin\omega t)$

Force needed along $\hat{\mathbf{i}}$ $= \frac{B_o d}{R}(\omega x\cos\omega t + v\sin\omega t)\times d\times B_o\sin\omega t$

$$= \frac{B_o^2 d^2}{R}(\omega x\cos\omega t + v\sin\omega t)\sin\omega t.$$

6.23 (i) Let the wire be at $x = x(t)$ at time t.

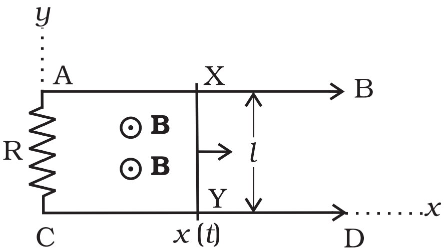
Flux = $B(t)\, l\, x(t)$

$$E = -\frac{d\phi}{dt} = -\frac{dB(t)}{dt}l\,x(t) - B(t)l.v(t)$$ (second term due to motional emf)

$$I = \frac{1}{R}E$$

$$\text{Force } = \frac{lB(t)}{R}\left[-\frac{dB}{dt}l\,x(t) - B(t)l\,v(t)\right]\hat{\mathbf{i}}$$

$$m\frac{d^2x}{dt^2} = -\frac{l^2B}{R}\frac{dB}{dt}x(t) - \frac{l^2B^2}{R}\frac{dx}{dt}$$



(ii) $\frac{dB}{dt} = 0, \qquad \frac{d^2x}{dt^2} + \frac{l^2B^2}{mR}\frac{dx}{dt} = 0$

$$\frac{dv}{dt} + \frac{l^2B^2}{mR}v = 0$$

$$v = A\exp\left(\frac{-l^2B^2t}{mR}\right)$$

At $t = 0$, $v = u$

$v(t) = u\exp(-l^2B^2t/mR)$.

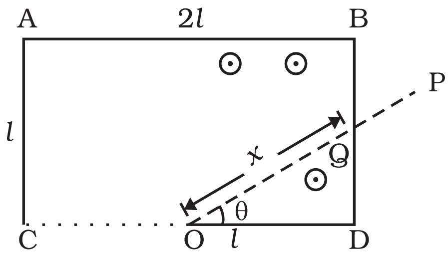
(iii) $I^2R = \frac{B^2l^2v^2(t)}{R^2} \times R = \frac{B^2l^2}{R}u^2\exp(-2l^2B^2t/mR)$

$$\text{Power lost } = \int_0^t I^2R\,dt = \frac{B^2l^2}{R}u^2\frac{mR}{2l^2B^2}\left[1 - e^{-(l^2B^2t/mR)}\right]$$

$$= \frac{m}{2}u^2 - \frac{m}{2}v^2(t)$$

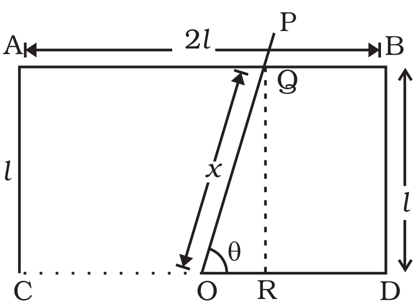
= decrease in kinetic energy.

6.24 Between time $t = 0$ and $t = \frac{\pi}{4\omega}$, the rod OP will make contact with the side BD. Let the length OQ of the contact at some time t such that $0 < t < \frac{\pi}{4\omega}$ be x. The flux through the area ODQ is



$$\phi = B\frac{1}{2}QD \times OD = B\frac{1}{2}l\tan\theta \times l$$

$$= \frac{1}{2}Bl^2\tan\theta \text{ where } \theta = \omega t$$



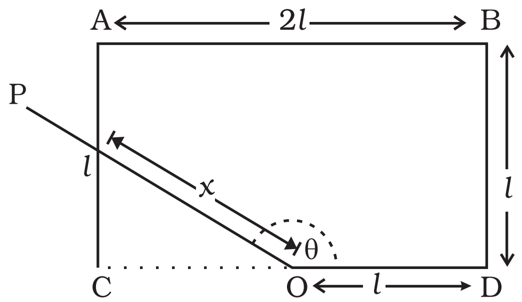
Thus the magnitude of the emf generated is $\varepsilon = \frac{d\phi}{dt} = \frac{1}{2}Bl^2\omega\sec^2\omega t$

The current is $I = \frac{\varepsilon}{R}$ where R is the resistance of the rod in contact.

$$R = \lambda x = \frac{\lambda l}{\cos \omega t}$$

$$\therefore \quad I = \frac{1}{2}\frac{Bl^2\omega}{\lambda l}\sec^2 \omega t \cos \omega t = \frac{Bl\omega}{2\lambda \cos \omega t}$$

For $\frac{\pi}{4\omega} < t < \frac{3\pi}{\omega}$ the rod is in contact with the side AB. Let the length of the rod in contact (OQ) be x. The flux through OQBD is $\phi = \left(l^2 + \frac{1}{2}\frac{l^2}{\tan \theta}\right)B$ where θ = ωt



Thus the magnitude of emf generated is

$$\varepsilon = \frac{d\phi}{dt} = \frac{1}{2}Bl^2\omega \frac{\sec^2 \omega t}{\tan^2 \omega t}$$

The current is $I = \frac{\varepsilon}{R} = \frac{\varepsilon}{\lambda x} = \frac{\varepsilon \sin \omega t}{\lambda l} = \frac{1}{2}\frac{Bl\omega}{\lambda \sin \omega t}$

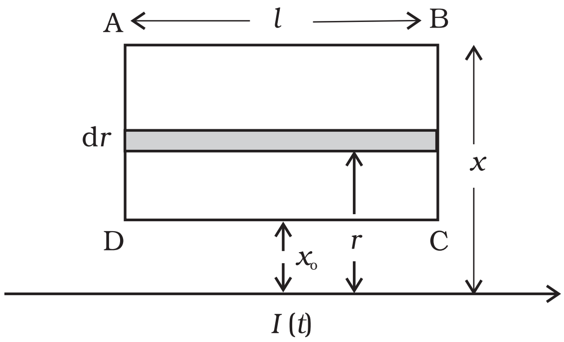
For $\frac{3\pi}{\omega} < t < \frac{\pi}{\omega}$ the rod will be in touch with OC. The Flux through OQABD is $\phi = \left(2l^2 - \frac{l^2}{2\tan \omega t}\right)B$

Thus the magnitude of emf

$$\varepsilon = \frac{d\phi}{dt} = \frac{B\omega l^2 \sec^2 \omega t}{2\tan^s \omega t}$$

$$I = \frac{\varepsilon}{R} = \frac{\varepsilon}{\lambda x} = \frac{1}{2}\frac{Bl\omega}{\lambda \sin \omega t}$$

6.25 At a distance r from the wire,



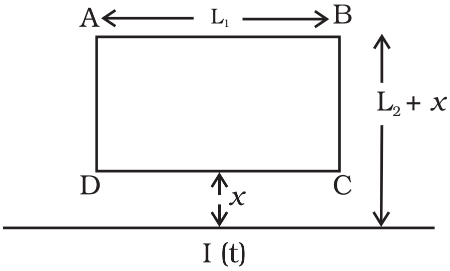
Field $B(r) = \frac{\mu_o I}{2\pi r}$ (out of paper).

Total flux through the loop is

$$\text{Flux} = \frac{\mu_o I}{2\pi} l \int_{x_o}^{x} \frac{dr}{r} = \frac{\mu_o I}{2\pi} l n \frac{x}{x_o}$$

$$\frac{1}{R}\frac{dI}{dt}=\frac{\varepsilon}{R}=I=\frac{\mu_o l}{2\pi}\frac{\lambda}{R}\ln\frac{x}{x_0}$$

6.26 If I (t) is the current in the loop.



$$I(t)=\frac{1}{R}\frac{d\phi}{dt}$$

If Q is the charge that passed in time t,

$$I(t)=\frac{dQ}{dt} \text{ or } \frac{dQ}{dt}=\frac{1}{R}\frac{d\phi}{dt}$$

Integrating $Q(t_1)-Q(t_2)=\frac{1}{R}\left[\phi(t_1)-\phi(t_2)\right]$

$$\phi(t_1)=L_1\frac{\mu_o}{2\pi}\int_x^{L_2+x}\frac{dx'}{x'}I(t_1)$$

$$=\frac{\mu_o L_1}{2\pi}I(t_1)\ln\frac{L_2+x}{x}$$

The magnitute of charge is

$$Q=\frac{\mu_o L_1}{2\pi}\ln\frac{L_2+x}{x}[I_o-0]$$

$$=\frac{\mu_o L_1 I_1}{2\pi}\ln\left(\frac{L_2+x}{x}\right).$$

6.27 $2\pi bE=E.M.F=\frac{B.\pi a^2}{\Lambda t}$ where E is the electric field generated around the ring.

$$\text{Torque} = b\times\text{Force} = Q\,E\,b = Q\left[\frac{B\pi a^2}{2\pi b\Delta t}\right]b$$

$$=Q\frac{Ba^2}{2\Delta t}$$

If ΔL is the change in angular momentum

$$\Delta L=\text{Torque}\times\Delta t=Q\frac{Ba^2}{2}$$

Initial angular momentum = 0

Final angular momentum $= mb^2\omega = \dfrac{QBa^2}{2}$

$$\omega = \frac{QBa^2}{2mb^2}.$$

6.28 $$m\frac{d^2x}{dt^2} = mg\sin\theta - \frac{B\cos\theta\, d}{R}\left(\frac{dx}{dt}\right)\times(Bd)\cos\theta$$

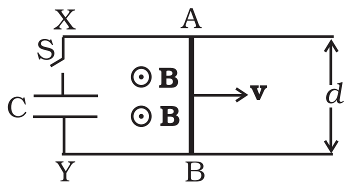
$$\frac{dv}{dt} = g\ \sin\theta - \frac{B^2d^2}{mR}(\cos\theta)^2\, v$$

$$\frac{dv}{dt} + \frac{B^2d^2}{mR}(\cos\theta)^2\, v = g\ \sin\theta$$

$$v = \frac{g\sin\theta}{\left(\dfrac{B^2d^2\cos^2\theta}{mR}\right)} + A\exp\left(-\frac{B^2d^2}{mR}(\cos^2\theta)t\right)$$ (A is a constant to be determine by initial conditions)

$$= \frac{mgR\sin\theta}{B^2d^2\cos^2\theta}\left(1 - \exp\left(-\frac{B^2d^2}{mR}(\cos^2\theta)t\right)\right)$$

6.29 If Q (t) is charge on the capacitor (note current flows from A to B)



$$I = \frac{vBd}{R} - \frac{Q}{RC}$$

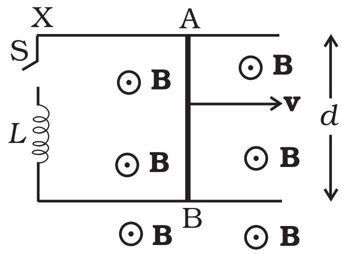
$$\Rightarrow \frac{Q}{RC} + \frac{dQ}{dt} = \frac{vBd}{R}$$

$$Q = vBdC + Ae^{-t/RC}$$

$$\therefore \Rightarrow Q = vBdC\left[1 - e^{-t/RC}\right]$$

(At time $t = 0$, $Q = 0 = A = -vBdc$). Differentiating, we get

$$I = \frac{vBd}{R}e^{-t/RC}$$



6.30 $$-L\frac{dI}{dt} + vBd = IR$$

$$L\frac{dI}{dt} + IR = vBd$$

$$I = \frac{vBd}{R} + A\,e^{-Rt/2}$$

At $t = 0$ $I = 0 \Rightarrow A = -\frac{vBd}{R}$

$$I = \frac{vBd}{R}\left(1 - e^{-Rt/L}\right).$$

6.31 $\frac{d\phi}{dt}$ = rate of change in flux = $(\pi l^2)\, B_o\, l\, \frac{dz}{dt} = IR$.

$$I = \frac{\pi l^2 B_o \lambda}{R} v$$

Energy lost/second = $I^2 R = \frac{(\pi l^2 \lambda)^2 B_o^2 v^2}{R}$

This must come from rate of change in PE = $m\,g\frac{dz}{dt} = mgv$

(as kinetic energy is constant for v= constant)

Thus, $mgv = \frac{(\pi l^2 \lambda B_0)^2 v^2}{R}$

Or, $v = \frac{mgR}{(\pi l^2 \lambda B_o)^2}$.

6.32 Magnetic field due to a solnoid S, $B = \mu_0 nI$

Magnetic flux in smaller coil $\phi = NBA$ where $A = \pi b^2$

So $e = \frac{-d\phi}{dt} = \frac{-d}{dt}(NBA)$

$$= -N\pi b^2 \frac{d(B)}{dt} = -N\pi b^2 \frac{d}{dt}(\mu_0 nI)$$

$$= -N\pi b^2 \mu_0 n \frac{dI}{dt}$$

$$= -Nn\pi\mu_0 b^2 \frac{d}{dt}(mt^2 + C) = -\mu_0 Nn\pi b^2 2mt$$

$$e = -\mu_0 Nn\pi b^2 2mt$$

Negative sign signifies opposite nature of induced emf. The magnitude of emf varies with time as shown in the Fig.