

# Physics

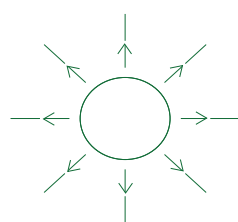
## NCERT Exemplar Problems

### Chapter 1

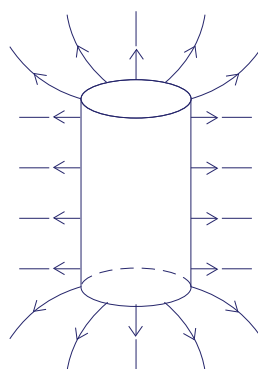
#### *Electric Charges and Fields*

- 1.1 (a) as  $q_1$  is negative, so  $Q$  will attract  $q_1$ .
- 1.2 (a) as electric field is moves +ve to -ve and always normal to the surface.
- 1.3 (d) as the net charge enclosed by all figures is same and flux =  $q/\epsilon$
- 1.4 (b) as  $q_2$  and  $q_4$  are enclosed by the Gaussian Surface.
- 1.5 (c) as the Electric Field is not uniform.
- 1.6 (a) directed perpendicular to the plane and away from the plane.
- 1.7 (a) perpendicular to the diameter
- 1.8 (c), (d)
- 1.9 (b), (d)
- 1.10 (b), (d)
- 1.11 (c), (d)
- 1.12 (a), (c).
- 1.13 (a), (b), (c) and (d).
- 1.14 Zero.
- 1.15 (i)  $\frac{-Q}{4\pi R_1^2}$  (ii)  $\frac{Q}{4\pi R_2^2}$
- 1.16 The electric fields bind the atoms to neutral entity. Fields are caused by excess charges. There can be no excess charge on the inter surface of an isolated conductor.
- 1.17 No, the field may be normal. However, the converse is true.

1.18



Top view



Side view

1.19 (i)  $\frac{q}{8\epsilon_0}$  (ii)  $\frac{q}{4\epsilon_0}$  (iii)  $\frac{q}{2\epsilon_0}$  (iv)  $\frac{q}{2\epsilon_0}$ .

1.20 1 Molar mass  $M$  of Al has  $N_A = 6.023 \times 10^{23}$  atoms.

$\therefore m =$  mass of Al paisa coin has  $N = N_A \frac{m}{M}$  atoms

Now,  $Z_{Al} = 13$ ,  $M_{Al} = 26.9815\text{g}$

Hence  $N = 6.02 \times 10^{23}$  atoms/mol  $\times \frac{0.75}{26.9815\text{g/mol}}$

$= 1.6733 \times 10^{22}$  atoms

$\therefore q =$  +ve charge in paisa  $= N Ze$

$= (1.67 \times 10^{22})(13) (1.60 \times 10^{-19}\text{C})$

$= 3.48 \times 10^4 \text{ C}.$

$q = 34.8 \text{ kC}$  of  $\pm$ ve charge.

This is an enormous amount of charge. Thus we see that ordinary neutral matter contains enormous amount of  $\pm$  charges.

1.21 (i)  $F_1 = \frac{|q|^2}{4\pi\epsilon_0 r_1^2} = \left(8.99 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}\right) \frac{(3.48 \times 10^4 \text{C})^2}{10^{-4} \text{m}^2} = 1.1 \times 10^{23} \text{N}$

(ii)  $\frac{F_2}{F_1} = \frac{r_1^2}{r_2^2} = \frac{(10^{-2} \text{m})^2}{(10^2 \text{m})^2} = 10^{-8} \Rightarrow F_2 = F_1 \times 10^{-8} = 1.1 \times 10^{15} \text{N}$

(iii)  $\frac{F_3}{F_1} = \frac{r_1^2}{r_3^2} = \frac{(10^{-2} \text{m})^2}{(10^6 \text{m})^2} = 10^{-16}$

$F_3 = 10^{-16} F_1 = 1.1 \times 10^7 \text{N}.$

**Conclusion:** When separated as point charges these charges exert an enormous force. It is not easy to disturb electrical neutrality.

- 1.22 (i) Zero, from symmetry.  
 (ii) Removing a +ve Cs ion is equivalent to adding singly charged -ve Cs ion at that location.  
 Net force then is

$$F = \frac{e^2}{4\pi\epsilon_0 r^2}$$

where  $r$  = distance between the Cl ion and a Cs ion.

$$= \sqrt{(0.20)^2 + (0.20)^2 + (0.20)^2} \times 10^{-9} = \sqrt{3(0.20)^2} \times 10^{-9}$$

$$= 0.346 \times 10^{-9} \text{ m}$$

$$\text{Hence, } F = \frac{(8.99 \times 10^9)(1.6 \times 10^{-19})^2}{(0.346 \times 10^{-9})^2} = 192 \times 10^{-11}$$

$$= 1.92 \times 10^{-9} \text{ N}$$

Ans  $1.92 \times 10^{-9} \text{ N}$ , directed from A to  $\text{Cl}^-$

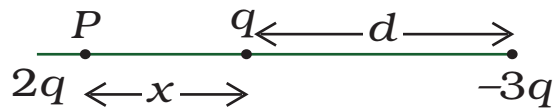
- 1.23 At P: on  $2q$ , Force due to  $q$  is to the left and that due to  $-3q$  is to the right.

$$\therefore \frac{2q^2}{4\pi\epsilon_0 x^2} = \frac{6q^2}{4\pi\epsilon_0 (d+x)^2}$$

$$\therefore (d+x)^2 = 3x^2$$

$$\therefore 2x^2 - 2dx - d^2 = 0$$

$$x = \frac{d \pm \sqrt{3d}}{2}$$



(-ve sign would be between  $q$  and  $-3q$  and hence is unacceptable.)

$$x = \frac{d}{2} + \frac{\sqrt{3d}}{2} = \frac{d}{2}(1 + \sqrt{3}) \text{ to the left of } q.$$

- 1.24 (a) Charges A and C are positive since lines of force emanate from them.  
 (b) Charge C has the largest magnitude since maximum number of field lines are associated with it.  
 (c) (i) near A. There is no neutral point between a positive and a negative charge. A neutral point may exist between two like charges. From the figure we see that a neutral point exists between charges A and C. Also between two like charges the neutral point is closer to the charge with smaller magnitude. Thus, electric field is zero near charge A.

- 1.25 (a) (i) zero      (ii)  $\frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$  along  $\overline{OA}$       (iii)  $\frac{1}{4\pi\epsilon_0} \frac{2q}{r^2}$  along  $\overline{OA}$

- (b) same as (a).

- 1.26 (a) Let the Universe have a radius  $R$ . Assume that the hydrogen atoms are uniformly distributed. The charge on each hydrogen atom is  $e_H = -(1 + y)e + e = -ye = |ye|$

The mass of each hydrogen atom is  $\sim m_p$  (mass of proton). Expansion starts if the Coulomb repulsion on a hydrogen atom, at  $R$ , is larger than the gravitational attraction.

Let the Electric Field at  $R$  be  $\mathbf{E}$ . Then

$$4\pi R^2 E = \frac{4}{3\epsilon_0} \pi R^3 N |ye| \quad (\text{Gauss's law})$$

$$\mathbf{E}(R) = \frac{1}{3} \frac{N|ye|}{\epsilon_0} R \hat{\mathbf{r}}$$

Let the gravitational field at  $R$  be  $G_R$ . Then

$$-4\pi R^2 G_R = 4\pi G m_p \frac{4}{3} \pi R^3 N$$

$$G_R = -\frac{4}{3} \pi G m_p N R$$

$$\mathbf{G}_R(\mathbf{R}) = -\frac{4}{3} \pi G m_p N R \hat{\mathbf{r}}$$

Thus the Coulombic force on a hydrogen atom at  $R$  is

$$ye\mathbf{E}(R) = \frac{1}{3} \frac{Ny^2e^2}{\epsilon_0} R \hat{\mathbf{r}}$$

The gravitational force on this atom is

$$m_p \mathbf{G}_R(R) = -\frac{4\pi}{3} GNm_p^2 R \hat{\mathbf{r}}$$

The net force on the atom is

$$\mathbf{F} = \left( \frac{1}{3} \frac{Ny^2e^2}{\epsilon_0} R - \frac{4\pi}{3} GNm_p^2 R \right) \hat{\mathbf{r}}$$

The critical value is when

$$\frac{1}{3} \frac{Ny_c^2e^2}{\epsilon_0} R = \frac{4\pi}{3} GNm_p^2 R$$

$$\Rightarrow y_c^2 = 4\pi\epsilon_0 G \frac{m_p^2}{e^2}$$

$$\square \frac{7 \times 10^{-11} \times 1.8^2 \times 10^6 \times 81 \times 10^{-62}}{9 \times 10^9 \times 1.6^2 \times 10^{-38}}$$

$$\square 63 \times 10^{-38}$$

$$\therefore y_c \square 8 \times 10^{-19} \square 10^{-18}$$

- (b) Because of the net force, the hydrogen atom experiences an acceleration such that

$$m_p \frac{d^2 R}{dt^2} = \left( \frac{1}{3} \frac{Ny^2 e^2}{e_o} R - \frac{4p}{3} GNm_p^2 R \right)$$

$$\text{Or, } \frac{d^2 R}{dt^2} = a^2 R \text{ where } a^2 = \frac{1}{m_p} \left( \frac{1}{3} \frac{Ny^2 e^2}{e_o} - \frac{4p}{3} GNm_p^2 \right)$$

This has a solution  $R = Ae^{at} + Be^{-at}$

As we are seeking an expansion,  $B = 0$ .

$$\therefore R = Ae^{at}$$

$$\Rightarrow \dot{R} = \alpha Ae^{at} = \alpha R$$

Thus, the velocity is proportional to the distance from the centre.

- 1.27 (a) The symmetry of the problem suggests that the electric field is radial. For points  $r < R$ , consider a spherical Gaussian surfaces. Then on the surface

$$\oiint \mathbf{E}_r \cdot d\mathbf{S} = \frac{1}{\epsilon_o} \int_V \rho dv$$

$$4\pi r^2 E_r = \frac{1}{\epsilon_o} 4\pi k \int_0^r r'^3 dr'$$

$$= \frac{1}{\epsilon_o} \frac{4\pi k}{4} r^4$$

$$\therefore E_r = \frac{1}{4\epsilon_o} kr^2$$

$$\mathbf{E}(r) = \frac{1}{4\epsilon_o} kr^2 \hat{\mathbf{r}}$$

For points  $r > R$ , consider a spherical Gaussian surfaces' of radius  $r$ ,

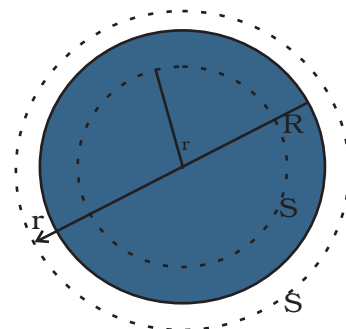
$$\oiint \mathbf{E}_r \cdot d\mathbf{S} = \frac{1}{\epsilon_o} \int_V \rho dv$$

$$4\pi r^2 E_r = \frac{4\pi k}{\epsilon_o} \int_0^R r^3 dr$$

$$= \frac{4\pi k}{\epsilon_o} \frac{R^4}{4}$$

$$\therefore E_r = \frac{k}{4\epsilon_o} \frac{R^4}{r^2}$$

$$\mathbf{E}(r) = (k / 4\epsilon_o) (R^4 / r^2) \hat{\mathbf{r}}$$

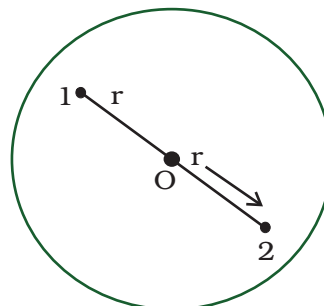


- (b) The two protons must be on the opposite sides of the centre along a diameter. Suppose the protons are at a distance  $r$  from the centre.

$$\text{Now, } 4\pi \int_0^R kr'^3 dr = 2e$$

$$\therefore \frac{4\pi k}{4} R^4 = 2e$$

$$\therefore k = \frac{2e}{\pi R^4}$$



Consider the forces on proton 1. The attractive force due to the charge distribution is

$$-e\mathbf{E}_r = -\frac{e}{4\epsilon_0} kr^2 \hat{\mathbf{r}} = -\frac{2e^2}{4\pi\epsilon_0} \frac{r^2}{R^4} \hat{\mathbf{r}}$$

The repulsive force is  $\frac{e^2}{4\pi\epsilon_0} \frac{1}{(2r)^2} \hat{\mathbf{r}}$

$$\text{Net force is } \left( \frac{e^2}{4\pi\epsilon_0 4r^2} - \frac{2e^2}{4\pi\epsilon_0} \frac{r^2}{R^4} \right) \hat{\mathbf{r}}$$

This is zero such that

$$\frac{e^2}{16\pi\epsilon_0 r^2} = \frac{2e^2}{4\pi\epsilon_0} \frac{r^2}{R^4}$$

$$\text{Or, } r^4 = \frac{4R^4}{32} = \frac{R^4}{8}$$

$$\Rightarrow r = \frac{R}{(8)^{1/4}}$$

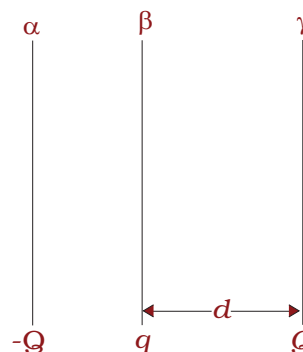
Thus, the protons must be at a distance  $r = \frac{R}{\sqrt[4]{8}}$  from the centre.

- 1.28 (a) The electric field at  $\gamma$  due to plate  $\alpha$  is  $-\frac{Q}{S2\epsilon_0} \hat{\mathbf{x}}$

The electric field at  $\gamma$  due to plate  $\beta$  is  $\frac{q}{S2\epsilon_0} \hat{\mathbf{x}}$

Hence, the net electric field is

$$\mathbf{E}_1 = \frac{(Q - q)}{2\epsilon_0 S} (-\hat{\mathbf{x}})$$



- (b) During the collision plates  $\beta$  &  $\gamma$  are together and hence must be at one potential. Suppose the charge on  $\beta$  is  $q_1$  and on  $\gamma$  is  $q_2$ . Consider a point O. The electric field here must be zero.

$$\text{Electric field at O due to } \alpha = -\frac{Q}{2\epsilon_0 S} \hat{\mathbf{x}}$$

Electric field at O due to  $\beta = -\frac{q_1}{2\epsilon_0 S} \hat{\mathbf{x}}$

Electric Field at O due to  $\gamma = -\frac{q_2}{2\epsilon_0 S} \hat{\mathbf{x}}$

$$\therefore \frac{-(Q+q_2)}{2\epsilon_0 S} + \frac{q_1}{2\epsilon_0 S} = 0$$

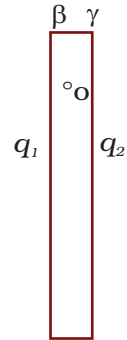
$$\Rightarrow q_1 - q_2 = Q$$

Further,  $q_1 + q_2 = Q + q$

$$\Rightarrow q_1 = Q + q/2$$

and  $q_2 = q/2$

Thus the charge on  $\beta$  and  $\gamma$  are  $Q + q/2$  and  $q/2$ , respectively.



- (c) Let the velocity be  $v$  at the distance  $d$  after the collision. If  $m$  is the mass of the plate  $\gamma$ , then the gain in K.E. over the round trip must be equal to the work done by the electric field.

After the collision, the electric field at  $\gamma$  is

$$\mathbf{E}_2 = -\frac{Q}{2\epsilon_0 S} \hat{\mathbf{x}} + \frac{(Q+q/2)}{2\epsilon_0 S} \hat{\mathbf{x}} = \frac{q/2}{2\epsilon_0 S} \hat{\mathbf{x}}$$

The work done when the plate  $\gamma$  is released till the collision is  $F_1 d$  where  $F_1$  is the force on plate  $\gamma$ .

The work done after the collision till it reaches  $d$  is  $F_2 d$  where  $F_2$  is the force on plate  $\gamma$ .

$$F_1 = E_1 Q = \frac{(Q-q)Q}{2\epsilon_0 S}$$

$$\text{and } F_2 = E_2 q/2 = \frac{(q/2)^2}{2\epsilon_0 S}$$

$\therefore$  Total work done is

$$\frac{1}{2\epsilon_0 S} [(Q-q)Q + (q/2)^2] d = \frac{1}{2\epsilon_0 S} (Q - q/2)^2 d$$

$$\Rightarrow (1/2)mv^2 = \frac{d}{2\epsilon_0 S} (Q - q/2)^2$$

$$\therefore v = (Q - q/2) \left( \frac{d}{m\epsilon_0 S} \right)^{1/2}$$

1.29 (i)  $F = \frac{Qq}{r^2} = 1 \text{ dyne} = \frac{[1 \text{esu of charge}]^2}{[1 \text{cm}]^2}$

Or,

$$1 \text{ esu of charge} = 1 (\text{dyne})^{1/2} (\text{cm})$$

$$\text{Hence, } [1 \text{ esu of charge}] = [F]^{1/2} L = [MLT^{-2}]^{1/2} L = M^{1/2} L^{3/2} T^{-1}$$

$$[1 \text{ esu of charge}] = M^{1/2} L^{3/2} T^{-1}$$

Thus charge in cgs unit is expressed as fractional powers (1/2) of M and (3/2) of L.

- (ii) Consider the coulomb force on two charges, each of magnitude 1 esu of charge separated by a distance of 1 cm:

The force is then 1 dyne =  $10^{-5}$  N.

This situation is equivalent to two charges of magnitude  $x$  C separated by  $10^{-2}$ m.

This gives:

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{x^2}{10^{-4}}$$

which should be 1 dyne =  $10^{-5}$  N. Thus

$$\frac{1}{4\pi\epsilon_0} \cdot \frac{x^2}{10^{-4}} = 10^{-5} \Rightarrow \frac{1}{4\pi\epsilon_0} = \frac{10^{-9}}{x^2} \frac{\text{Nm}^2}{\text{C}^2}$$

With  $x = \frac{1}{[3] \times 10^9}$ , this yields

$$\frac{1}{4\pi\epsilon_0} = 10^{-9} \times [3]^2 \times 10^{18} = [3]^2 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}$$

With  $[3] \rightarrow 2.99792458$ , we get

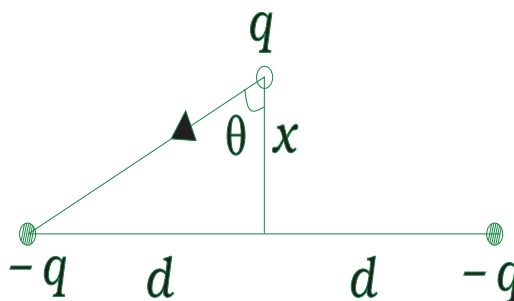
$$\frac{1}{4\pi\epsilon_0} = 8.98755 \dots \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \text{ exactly}$$

### 1.30 Net force $F$ on $q$ towards the centre $O$

$$F = 2 \frac{q^2}{4\pi\epsilon_0 r^2} \cos \theta = -\frac{2q^2}{4\pi\epsilon_0 r^2} \cdot \frac{x}{r}$$

$$F = \frac{-2q^2}{4\pi\epsilon_0} \frac{x}{(d^2 + x^2)^{3/2}}$$

$$\approx \frac{-2q^2}{4\pi\epsilon_0 d^3} x = -kx \text{ for } x \ll d.$$



Thus, the force on the third charge  $q$  is proportional to the displacement and is towards the centre of the two other charges. Therefore, the motion of the third charge is harmonic with frequency

$$\omega = \sqrt{\frac{2q^2}{4\pi\epsilon_0 d^3 m}} = \sqrt{\frac{k}{m}}$$

$$\text{and hence } T = \frac{2\pi}{\omega} \left[ \frac{8\pi^3 \epsilon_0 m d^3}{q^2} \right]^{1/2}.$$

- 1.31 (a) Slight push on  $q$  along the axis of the ring gives rise to the situation shown in Fig (b). A and B are two points on the ring at the end of a diameter.



Force on  $q$  due to line elements  $\frac{-Q}{2\pi R}$  at A and B is

$$F_{A+B} = 2 \cdot \frac{-Q}{2\pi R} \cdot q \cdot \frac{1}{4\pi\epsilon_0} \cdot \frac{1}{r^2} \cdot \cos\theta$$

$$= \frac{-Qq}{\pi R \cdot 4\pi\epsilon_0} \cdot \frac{1}{(z^2 + R^2)} \cdot \frac{z}{(z^2 + R^2)^{1/2}}$$

Total force due to ring on  $q = (F_{A+B})(\pi R)$

$$= \frac{-Qq}{4\pi\epsilon_0} \frac{z}{(z^2 + R^2)^{3/2}}$$

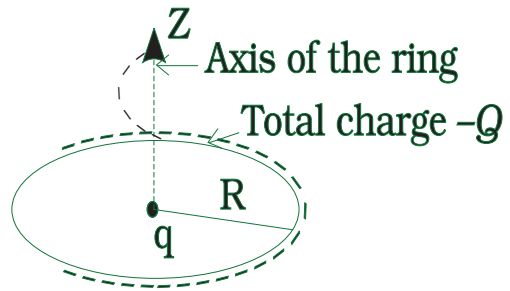
□  $\frac{-Qq}{4\pi\epsilon_0}$  for  $z \ll R$

Thus, the force is proportional to negative of displacement. Motion under such forces is harmonic.

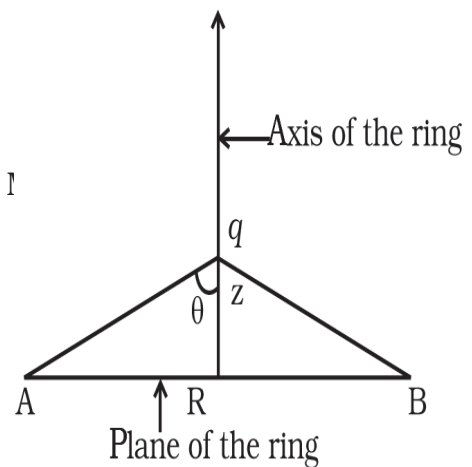
(b) From (a)

$$m \frac{d^2z}{dt^2} = -\frac{Qqz}{4\pi\epsilon_0 R^3} \text{ or } \frac{d^2z}{dt^2} = -\frac{Qq}{4\pi\epsilon_0 m R^3} z$$

That is,  $\omega^2 = \frac{Qq}{4\pi\epsilon_0 m R^3}$ . Hence  $T = 2\pi \sqrt{\frac{4\pi\epsilon_0 m R^3}{Qq}}$



(a)



(b)