

Mathematics

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(Chapter 5)(Continuity and Differentiability)

XII

Exercise 5.4

Question 1:

Differentiate the following w.r.t. x : $\frac{e^x}{\sin x}$

Answer

$$\text{Let } y = \frac{e^x}{\sin x}$$

By using the quotient rule, we obtain

$$\begin{aligned}\frac{dy}{dx} &= \frac{\sin x \frac{d}{dx}(e^x) - e^x \frac{d}{dx}(\sin x)}{\sin^2 x} \\ &= \frac{\sin x \cdot (e^x) - e^x \cdot (\cos x)}{\sin^2 x} \\ &= \frac{e^x (\sin x - \cos x)}{\sin^2 x}, x \neq n\pi, n \in \mathbf{Z}\end{aligned}$$

Question 2:

Differentiate the following w.r.t. x : $e^{\sin^{-1} x}$

Answer

$$\text{Let } y = e^{\sin^{-1} x}$$

By using the chain rule, we obtain

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(e^{\sin^{-1} x}) \\ \Rightarrow \frac{dy}{dx} &= e^{\sin^{-1} x} \cdot \frac{d}{dx}(\sin^{-1} x) \\ &= e^{\sin^{-1} x} \cdot \frac{1}{\sqrt{1-x^2}} \\ &= \frac{e^{\sin^{-1} x}}{\sqrt{1-x^2}} \\ \therefore \frac{dy}{dx} &= \frac{e^{\sin^{-1} x}}{\sqrt{1-x^2}}, x \in (-1, 1)\end{aligned}$$



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Question 3:

Differentiate the following w.r.t. x :

$$e^{x^3}$$

Answer

$$\text{Let } y = e^{x^3}$$

By using the chain rule, we obtain

$$\frac{dy}{dx} = \frac{d}{dx}(e^{x^3}) = e^{x^3} \cdot \frac{d}{dx}(x^3) = e^{x^3} \cdot 3x^2 = 3x^2 e^{x^3}$$

Question 4:

Differentiate the following w.r.t. x :

$$\sin(\tan^{-1} e^{-x})$$

Answer

$$\text{Let } y = \sin(\tan^{-1} e^{-x})$$

By using the chain rule, we obtain

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx}[\sin(\tan^{-1} e^{-x})] \\ &= \cos(\tan^{-1} e^{-x}) \cdot \frac{d}{dx}(\tan^{-1} e^{-x}) \\ &= \cos(\tan^{-1} e^{-x}) \cdot \frac{1}{1+(e^{-x})^2} \cdot \frac{d}{dx}(e^{-x}) \\ &= \frac{\cos(\tan^{-1} e^{-x})}{1+e^{-2x}} \cdot e^{-x} \cdot \frac{d}{dx}(-x) \\ &= \frac{e^{-x} \cos(\tan^{-1} e^{-x})}{1+e^{-2x}} \times (-1) \\ &= \frac{-e^{-x} \cos(\tan^{-1} e^{-x})}{1+e^{-2x}} \end{aligned}$$

Question 5:

Differentiate the following w.r.t. x :



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$$\log(\cos e^x)$$

Answer

$$\text{Let } y = \log(\cos e^x)$$

By using the chain rule, we obtain

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}[\log(\cos e^x)] \\ &= \frac{1}{\cos e^x} \cdot \frac{d}{dx}(\cos e^x) \\ &= \frac{1}{\cos e^x} \cdot (-\sin e^x) \cdot \frac{d}{dx}(e^x) \\ &= \frac{-\sin e^x}{\cos e^x} \cdot e^x \\ &= -e^x \tan e^x, e^x \neq (2n+1)\frac{\pi}{2}, n \in \mathbf{N}\end{aligned}$$

Question 6:

Differentiate the following w.r.t. x :

$$e^x + e^{x^2} + \dots + e^{x^5}$$

Answer

$$\begin{aligned}\frac{d}{dx}(e^x + e^{x^2} + \dots + e^{x^5}) \\ &= \frac{d}{dx}(e^x) + \frac{d}{dx}(e^{x^2}) + \frac{d}{dx}(e^{x^3}) + \frac{d}{dx}(e^{x^4}) + \frac{d}{dx}(e^{x^5}) \\ &= e^x + \left[e^{x^2} \times \frac{d}{dx}(x^2) \right] + \left[e^{x^3} \cdot \frac{d}{dx}(x^3) \right] + \left[e^{x^4} \cdot \frac{d}{dx}(x^4) \right] + \left[e^{x^5} \cdot \frac{d}{dx}(x^5) \right] \\ &= e^x + (e^{x^2} \times 2x) + (e^{x^3} \times 3x^2) + (e^{x^4} \times 4x^3) + (e^{x^5} \times 5x^4) \\ &= e^x + 2xe^{x^2} + 3x^2e^{x^3} + 4x^3e^{x^4} + 5x^4e^{x^5}\end{aligned}$$

Question 7:

Differentiate the following w.r.t. x :

$$\sqrt{e^{\sqrt{x}}}, x > 0$$



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Answer

Let

$$y = \sqrt{e^{\sqrt{x}}}$$

$$y^2 = e^{\sqrt{x}}$$

By differentiating this relationship with respect to x , we obtain

$$y^2 = e^{\sqrt{x}}$$

$$\Rightarrow 2y \frac{dy}{dx} = e^{\sqrt{x}} \frac{d}{dx}(\sqrt{x}) \quad [\text{By applying the chain rule}]$$

$$\Rightarrow 2y \frac{dy}{dx} = e^{\sqrt{x}} \frac{1}{2} \cdot \frac{1}{\sqrt{x}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^{\sqrt{x}}}{4y\sqrt{x}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^{\sqrt{x}}}{4\sqrt{e^{\sqrt{x}}}\sqrt{x}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^{\sqrt{x}}}{4\sqrt{xe^{\sqrt{x}}}}, x > 0$$

Question 8:

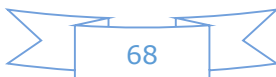
Differentiate the following w.r.t. x :

$$\log(\log x), x > 1$$

Answer

Let $y = \log(\log x)$

By using the chain rule, we obtain



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$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} [\log(\log x)] \\ &= \frac{1}{\log x} \cdot \frac{d}{dx} (\log x) \\ &= \frac{1}{\log x} \cdot \frac{1}{x} \\ &= \frac{1}{x \log x}, x > 1\end{aligned}$$

Question 9:

Differentiate the following w.r.t. x :

$$\frac{\cos x}{\log x}, x > 0$$

Answer

$$\text{Let } y = \frac{\cos x}{\log x}$$

By using the quotient rule, we obtain

$$\begin{aligned}\frac{dy}{dx} &= \frac{\frac{d}{dx}(\cos x) \times \log x - \cos x \times \frac{d}{dx}(\log x)}{(\log x)^2} \\ &= \frac{-\sin x \log x - \cos x \times \frac{1}{x}}{(\log x)^2} \\ &= \frac{-[x \log x \cdot \sin x + \cos x]}{x(\log x)^2}, x > 0\end{aligned}$$

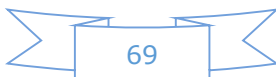
Question 10:

Differentiate the following w.r.t. x :

$$\cos(\log x + e^x), x > 0$$

Answer

$$\text{Let } y = \cos(\log x + e^x)$$



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By using the chain rule, we obtain

$$\begin{aligned}\frac{dy}{dx} &= -\sin(\log x + e^x) \cdot \frac{d}{dx}(\log x + e^x) \\ &= -\sin(\log x + e^x) \cdot \left[\frac{d}{dx}(\log x) + \frac{d}{dx}(e^x) \right] \\ &= -\sin(\log x + e^x) \cdot \left(\frac{1}{x} + e^x \right) \\ &= -\left(\frac{1}{x} + e^x \right) \sin(\log x + e^x), x > 0\end{aligned}$$

