Mathematics

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(Chapter 4)(Determinants)

XII

Exercise 4.1

Question 1:

Evaluate the determinants in Exercises 1 and 2.

$$\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$$

Answer

$\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$ = 2(−1) − 4(−5) = − 2 + 20 = 18

Question 2:

Evaluate the determinants in Exercises 1 and 2.

(i) $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$ (ii) $\begin{vmatrix} x^2-x+1 & x-1 \\ x+1 & x+1 \end{vmatrix}$

Answer

$\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$

(i) = (cos θ)(cos θ) − (−sin θ)(sin θ) = $\cos^2\theta + \sin^2\theta$ = 1

$\begin{vmatrix} x^2-x+1 & x-1 \\ x+1 & x+1 \end{vmatrix}$

(ii)

= $(x^2 - x + 1)(x + 1) - (x - 1)(x + 1)$

= $x^3 - x^2 + x + x^2 - x + 1 - (x^2 - 1)$

= $x^3 + 1 - x^2 + 1$

= $x^3 - x^2 + 2$

Question 3:

If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then show that $|2A| = 4|A|$

Answer

The given matrix is $A=\begin{bmatrix}1 & 2\\ 4 & 2\end{bmatrix}$.

$\therefore 2A = 2\begin{bmatrix}1 & 2\\ 4 & 2\end{bmatrix} = \begin{bmatrix}2 & 4\\ 8 & 4\end{bmatrix}$

$\therefore \text{L.H.S.} = |2A| = \begin{vmatrix}2 & 4\\ 8 & 4\end{vmatrix} = 2\times4-4\times8 = 8-32 = -24$

Now, $|A| = \begin{vmatrix}1 & 2\\ 4 & 2\end{vmatrix} = 1\times2-2\times4 = 2-8 = -6$

$\therefore \text{R.H.S.} = 4|A| = 4\times(-6) = -24$

$\therefore \text{L.H.S.} = \text{R.H.S.}$

Question 4:

If $A=\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 2\\ 0 & 0 & 4\end{bmatrix}$, then show that $|3A| = 27|A|$.

Answer

The given matrix is $A=\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 2\\ 0 & 0 & 4\end{bmatrix}$.

It can be observed that in the first column, two entries are zero. Thus, we expand along the first column (C_1) for easier calculation.

$$|A| = 1\begin{vmatrix}1 & 2\\0 & 4\end{vmatrix} - 0\begin{vmatrix}0 & 1\\0 & 4\end{vmatrix} + 0\begin{vmatrix}0 & 1\\1 & 2\end{vmatrix} = 1(4-0) - 0 + 0 = 4$$

$$\therefore 27|A| = 27(4) = 108 \qquad \text{...(i)}$$

$$\text{Now, } 3A = 3\begin{bmatrix}1 & 0 & 1\\0 & 1 & 2\\0 & 0 & 4\end{bmatrix} = \begin{bmatrix}3 & 0 & 3\\0 & 3 & 6\\0 & 0 & 12\end{bmatrix}$$

$$\therefore |3A| = 3\begin{vmatrix}3 & 6\\0 & 12\end{vmatrix} - 0\begin{vmatrix}0 & 3\\0 & 12\end{vmatrix} + 0\begin{vmatrix}0 & 3\\3 & 6\end{vmatrix}$$

$$= 3(36-0) = 3(36) = 108 \qquad \text{...(ii)}$$

From equations (i) and (ii), we have:

$$|3A| = 27|A|$$

Hence, the given result is proved.

Question 5:

Evaluate the determinants

(i) $\begin{vmatrix}3 & -1 & -2\\0 & 0 & -1\\3 & -5 & 0\end{vmatrix}$ (iii) $\begin{vmatrix}3 & -4 & 5\\1 & 1 & -2\\2 & 3 & 1\end{vmatrix}$

(ii) $\begin{vmatrix}0 & 1 & 2\\-1 & 0 & -3\\-2 & 3 & 0\end{vmatrix}$ (iv) $\begin{bmatrix}2 & -1 & -2\\0 & 2 & -1\\3 & -5 & 0\end{bmatrix}$

Answer

(i) Let $A = \begin{vmatrix}3 & -1 & -2\\0 & 0 & -1\\3 & -5 & 0\end{vmatrix}$.

It can be observed that in the second row, two entries are zero. Thus, we expand along the second row for easier calculation.

$$|A| = -0\begin{vmatrix}-1 & -2\\-5 & 0\end{vmatrix} + 0\begin{vmatrix}3 & -2\\3 & 0\end{vmatrix} - (-1)\begin{vmatrix}3 & -1\\3 & -5\end{vmatrix} = (-15+3) = -12$$

(ii) Let $A = \begin{bmatrix}3 & -4 & 5\\1 & 1 & -2\\2 & 3 & 1\end{bmatrix}$.

By expanding along the first row, we have:

$$|A| = 3\begin{vmatrix}1 & -2\\3 & 1\end{vmatrix} + 4\begin{vmatrix}1 & -2\\2 & 1\end{vmatrix} + 5\begin{vmatrix}1 & 1\\2 & 3\end{vmatrix}$$
$$= 3(1+6) + 4(1+4) + 5(3-2)$$
$$= 3(7) + 4(5) + 5(1)$$
$$= 21 + 20 + 5 = 46$$

(iii) Let $A = \begin{bmatrix}0 & 1 & 2\\-1 & 0 & -3\\-2 & 3 & 0\end{bmatrix}$.

By expanding along the first row, we have:

$$|A| = 0\begin{vmatrix}0 & -3\\3 & 0\end{vmatrix} - 1\begin{vmatrix}-1 & -3\\-2 & 0\end{vmatrix} + 2\begin{vmatrix}-1 & 0\\-2 & 3\end{vmatrix}$$
$$= 0 - 1(0-6) + 2(-3-0)$$
$$= -1(-6) + 2(-3)$$
$$= 6 - 6 = 0$$

(iv) Let $A = \begin{bmatrix}2 & -1 & -2\\0 & 2 & -1\\3 & -5 & 0\end{bmatrix}$.

By expanding along the first column, we have:

$$|A| = 2\begin{vmatrix}2 & -1\\-5 & 0\end{vmatrix} - 0\begin{vmatrix}-1 & -2\\-5 & 0\end{vmatrix} + 3\begin{vmatrix}-1 & -2\\2 & -1\end{vmatrix}$$
$$= 2(0-5) - 0 + 3(1+4)$$
$$= -10 + 15 = 5$$

Question 6:

If $A=\begin{bmatrix}1 & 1 & -2\\ 2 & 1 & -3\\ 5 & 4 & -9\end{bmatrix}$, find $|A|$.

Answer

Let $A=\begin{bmatrix}1 & 1 & -2\\ 2 & 1 & -3\\ 5 & 4 & -9\end{bmatrix}$.

By expanding along the first row, we have:

$$\begin{aligned}|A| &= 1\begin{vmatrix}1 & -3\\ 4 & -9\end{vmatrix}-1\begin{vmatrix}2 & -3\\ 5 & -9\end{vmatrix}-2\begin{vmatrix}2 & 1\\ 5 & 4\end{vmatrix}\\ &=1(-9+12)-1(-18+15)-2(8-5)\\ &=1(3)-1(-3)-2(3)\\ &=3+3-6\\ &=6-6\\ &=0\end{aligned}$$

Question 7:

Find values of x, if

(i) $\begin{vmatrix}2 & 4\\ 2 & 1\end{vmatrix}=\begin{vmatrix}2x & 4\\ 6 & x\end{vmatrix}$ (ii) $\begin{vmatrix}2 & 3\\ 4 & 5\end{vmatrix}=\begin{vmatrix}x & 3\\ 2x & 5\end{vmatrix}$

Answer

(i) $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$

$\Rightarrow 2\times1-5\times4=2x\times x-6\times4$

$\Rightarrow 2-20=2x^2-24$

$\Rightarrow 2x^2=6$

$\Rightarrow x^2=3$

$\Rightarrow x=\pm\sqrt{3}$

(ii) $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

$\Rightarrow 2\times5-3\times4=x\times5-3\times2x$

$\Rightarrow 10-12=5x-6x$

$\Rightarrow -2=-x$

$\Rightarrow x=2$

Question 8:

If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, then x is equal to (A) 6 (B) ±6 (C) −6 (D) 0

Answer

Answer: B

$\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$

$\Rightarrow x^2-36=36-36$

$\Rightarrow x^2-36=0$

$\Rightarrow x^2=36$

$\Rightarrow x=\pm6$

Hence, the correct answer is B.